PHYSICAL AND MATHEMATICAL SCIENCES

UDC 517.956 DOI 10.23683/0321-3005-2018-2-6-11

THE CORRECTNESS OF THE DIRICHLET PROBLEM FOR ONECLASS OF DEGENERATE MULTIDIMENSIONAL ELLIPTIC-PARABOLIC EQUATIONS

***S.A. Aldashev*[1]**
[1]*Abay Kazakh Pedagogical National University, Almaty, Kazakhstan*

Serik A. Aldashev - Doctor of Physics and Mathematics, Professor, Department of Mathematics and Mathematical Modeling, Institute of Mathematics, Physics and Informatics, Abay Kazakh National Pedagogical University, Tole bi St., 86, Almaty, 050012, Kazakhstan, e-mail: aldash51@mail.ru

The correctness of boundary value problems on the plane for elliptic equations by the method of the theory of analytic functions of a complex variable has been well studied. When investigating similar questions, when the number of independent variables is greater than two, problems of a fundamental nature arise. A very attractive and convenient method of singular integral equations loses their validity due to the absence of any full theory of multidimensional singular integral equations. Boundary value problems for second-order elliptic equations in domains with edges have been studied in detail.

In the author's papers explicit forms of classical solutions of Dirichlet problems in cylindrical domains for multidimensional elliptic equations are found. In this paper we use the method proposed in the author's works, we show the unique solvability and obtain an explicit form of the classical solution of the Dirichlet problem in a cylindrical domain for one class of degenerate multidimensional elliptic-parabolic equations.

Keywords: *correctness, multidimensional degenerate equations, Dirichlet problem, spherical functions, Bessel function.*

References

1. Fikera G. K edinoi teorii kraevykh zadach dlya elliptiko-parabolicheskikh uravnenii vtorogo poryadka [On a unified theory of boundary-value problems for second-order elliptic-parabolic equations]. *Matematika*. 1963, vol. 7, No. 6, pp. 99-121.
2. Oleinik O.A., Radkevich E.V. *Uravneniya s neotritsatel'noi kharakteristicheskoi formoi* [Equations with a nonnegative characteristic form]. Moscow: Mosk. un-ta, 2010, 360 p.
3. Aldashev S.A. Korrektnost' zadachi Dirikhle v tsilindricheskoi oblasti dlya odnogo klassa mnogomernykh ellipticheskikh uravnenii [Correctness of the Dirichlet problem in a cylindrical domain for a class of multidimensional elliptic equations]. *Vestn. NGU.* Ser. mat., mekh., inf. 2012, vol. 12, iss. 1, pp. 7-13.
4. Aldashev S.A. Korrektnost' zadachi Dirikhle v tsilindricheskoi oblasti dlya vyrozhdayushchikhsya mnogomernykh ellipticheskikh uravnenii [Correctness of the Dirichlet problem in a cylindrical domain for degenerate multidimensional elliptic equations]. *Mat. zametki*. 2013, vol. 94, iss. 6, pp. 936-939.
5. Aldashev S.A. Korrektnost' zadachi Dirikhle v tsilindricheskoi oblasti dlya mnogomernogo elliptiko-parabolicheskogo uravneniya [Correctness of the Dirichlet problem in a cylindrical domain for a multidimensional elliptic-parabolic equation]. *Izv. Sarat. un-ta.* Ser. mat., mekh., inf. 2014, vol. 14, iss. 1, pp. 5-10.
6. Aldashev S.A. Korrektnost' zadachi Dirikhle dlya odnogo klassa mnogomernykh elliptiko-parabolicheskikh uravnenii [Correctness of the Dirichlet problem for a class of multidimensional elliptic-parabolic equations]. *Izv. Sarat. un-ta.* Ser. mat., mekh., inf. 2016, vol. 16, iss. 2, pp. 125-132.
7. Mikhlin S.G. *Mnogomernye singulyarnye integraly i integral'nye uravneniya* [Multidimensional singular integrals and integral equations]. Moscow: Fizmatgiz, 1962, 254 p.
8. Aldashev S.A. *Kraevye zadachi dlya mnogomernykh giperbolicheskikh i smeshannykh uravnenii* [Boundary value problems for multidimensional hyperbolic and mixed equations]. Almaty: Gylym, 1994, 170 p.
9. Aldashev S.A. *Vyrozhdayushchiesya mnogomernye giperbolicheskie uravneniya* [Degenerate multidimensional hyperbolic equations]. Oral: ZKATU, 2007, 139 p.
10. Kamke E. *Spravochnik po obyknovennym differentsial'nym uravneniyam* [Handbook of ordinary differential equations]. Moscow: Nauka, 1965, 703 p.
11. Beitmen G., Erdeii A. *Vysshie transtsendentnye funktsii* [Higher transcendental functions]: in 2 vol. Moscow: Nauka, 1974, vol. 2, 297 p.

12. Kolmogorov A.N., Fomin S.V. *Elementy teorii funktsii i funktsional'nogo analiza* [Elements of the theory of functions and functional analysis]. Moscow: Nauka, 1976, 543 p.
13. Tikhonov A.N., Samarskii A.A. *Uravneniya matematicheskoi fiziki* [Equations of mathematical physics]. Moscow: Nauka, 1966, 724 p.
14. Smirnov V.I. *Kurs vysshei matematiki* [The course of higher mathematics]. Moscow: Nauka, 1981, vol. 4, 550 p.
15. Fridman A. *Uravneniya s chastnymi proizvodnymi parabolicheskogo tipa* [Partial differential equations of parabolic type]. Moscow: Mir, 1968, 527 p.
16. Bers L., Dzhon F., Shekhter M. *Uravneniya s chastnymi proizvodnymi* [Partial differential equations]. Moscow: Mir, 1966, 352 p.
17. Aldashev S.A. Korrektnost' zadachi Dirikhle v tsilindricheskoi oblasti dlya vyrozhdayushchegosya mnogomernogo elliptiko-parabolicheskogo uravneniya [Correctness of the Dirichlet problem in a cylindrical domain for a degenerate multidimensional elliptic-parabolic equation]. *Zhurn. vychisl. i prikl. matematiki. KNUim. T. Shevchenko.* 2014, No. 3 (117), pp. 17-22.

Received ***October 26, 2017***

UDC 517.941 DOI 10.23683/0321-3005-2018-2-12-15

REGULAR BOUNDARY VALUE PROBLEM FOR THE DIFFERENTIAL BEAM ABOUT 15 WITH FIVEFOLD CHARACTERISTICS

A.I. Vagabov[1]
[1]Dagestan State University, Makhachkala, Russia

Abdulvagab I. Vagabov - Doctor of Physics and Mathema-tics, Professor, Department of Mathematic Analysis, Dagestan State University, Gadzhieva St., 43a, Makhachkala, Republic Dagestan, 367000, Russia, e-mail: algebra-dgu@mail.ru

The article is a continuation of works instances of two differential beams with the same n-fold and, respectively, with two three-time characteristics. The basis property of the root functions is considered for arbitrary decaying boundary conditions at the ends of the given interval. Problematic remains the question of the transfer of the corresponding theorems for the case of beams with three or more multiple characteristic roots. We are given a positive answer in the case of the beam 15 with three different characteristics. We note that our approach in this task will extend to broader classes of bundles with multiple characteristics.

We note the significant difference of the task from regular in the sense of G. Birkhoff and J.D. Tamarkin. On the one hand, significant differences of the characteristic roots of the primary differential operator are previously required. On the other, we have solved the problem with decaying boundary conditions, all of which is set at one end, with the exception of two related to the second end. Given the construction of the resolvent of the beam as meromorphic functions of the parameter λ. In the main theorem is proved that the full deduction option from the resolvent applied to 15 times differentiable functions (the derivative zero at the ends of the interval) is equal to this function. The specified deduction is a Fourier series in root functions of the original problem.

Keywords: *Cauchy function, multiple roots, Green function, Fourier series.*

References

1. Vagabov A.I. n-kratnaya formula razlozheniya v ryady Fur'e po kornevym elementam differentsial'nogo puchka s n-kratnoi kharakteristikoi [n-multiple expansion formula in Fourien series on root elements of a differential buch with the n-multiple characteristic]. *Dif. uravneniya.* 2016, vol. 52, No. 5, pp. 555-560.
2. Vagabov A.I. O bazisnosti sobstvennykh elementov differentsial'nogo puchka shestogo poryadka s trekhkratnymi kharakteristikami [On the basis property of the eigenelements of a sixth-order differential pencil with triple characteristics]. *Izv. vuzov. Sev.-Kavk. region. Estestv. nauki*. 2017, No. 1, pp. 14-17.
3. Naimark M.A. *Lineinye differentsial'nye operatory* [Linear differential operators]. Moscow: Nauka, 1969, 526 p.

Received ***February 20, 2018***

UDC 539.3 DOI 10.23683/0321-3005-2018-2-16-22

ON THE RECONSTRUCTION OF CHARACTERISTICS OF INCLUSION BASED ON THE TIMOSHENKO MODEL

***A.O. Vatulyan*[1,2], *K.V. Guschina*[1]**

[1]Southern Federal University, Rostov-on-Don, Russia,
[2]Southern Mathematical Institute - Branch of the Vladikavkaz Scientific Center, Russian Academy of Sciences, Vladikavkaz, Russia

Alexander O. Vatulyan - Doctor of Physics and Mathema- tics, Professor, Head of the Department of Elasticity Theory, Vorovich Institute of Mathematics, Mechanics and Computer Sciences, Southern Federal University, Milchakova St., 8a, Rostov-on-Don, 344090, Russia; Head of the Department of Differential Equations, Southern Mathematical Institute - Branch of the Vladikavkaz Scientific Center, Russian Academy of Sciences, Marcusa St., 22, Vladikavkaz, Republic of North Ossetia - Alania, 362027, Russia, e-mail: vatulyan@math.sfedu.ru

Ksenia V. Guschina - Student, Department of Elasticity Theory, Vorovich Institute of Mathematics, Mechanics and Computer Sciences, Southern Federal University, Milchakova St., 8a, Rostov-on-Don, 344090, Russia; e-mail: guschina.ksu@gmail.com

The oscillations of an elastic homogeneous beam for the Timoshenko model and a beam with an inclusion-type defect that has an ellipsoidal shape are studied. The adduction is carried out to the dimensionless form of the problem and the composition of the canonical system of equations. The physical characteristics of a non-uniform beam differ from the characteristics of a beam without a defect. The resonance values of a homogeneous beam are obtained and the influence of the inclusion parameters on the resonance values of a beam with a defect on the basis of the method of alignment is investigated. Using the methods of perturbation theory, a formula is constructed for corrections to the resonant frequencies, which relates resonance values for an intact beam and a beam with a defect. On the basis of the obtained relation, two inverse problems are solved. In the first, a step-by-step method for restoring the geometric characteristics of the inclusion of an ellipsoidal shape is implemented. In the second problem, the physical characteristics of an inhomogeneous beam are reconstructed, namely the ratio of the stiffness and density modules. The high accuracy of the proposed method is demonstrated.

Keywords: *oscillations, beam, inclusion, resonant frequency, inverse problem, correction, Timoshenko model, bending.*

References

1. Vatul'yan A.O., Soluyanov N.O. Identifikatsiya polosti v uprugom sterzhne pri analize poperechnykh kolebanii [Identification of a cavity in an elastic rod in the analysis of transverse oscillations]. *Prikladnaya mekhanika i tekhnicheskaya fizika.* 2008, vol. 49, No. 6, pp. 1015-1020.
2. Akhtyamov A.M., Satyev E.I. Opredelenie mestopolozheniya i ob"ema polosti v uprugom sterzhne po dvum sobstvennym chastotam ego kolebanii [Determination of the location and volume of a cavity in an elastic rod at two natural frequencies of its oscillations]. *Defektoskopiya*. 2012, No. 5, pp. 78-83.
3. Vatul'yan A.O., Osipov A.V. Ob odnom podkhode pri opredelenii parametrov defekta v balke [On one approach in determining the parameters of a defect in a beam]. *Defektoskopiya*. 2014, No. 11, pp. 37-47.
4. Shifrin E.I. Identifikatsiya ellipsoidal'nogo defekta v uprugom tele po rezul'tatam odnogo ispytaniya na odnoosnoe rastyazhenie (szhatie) [Identification of an ellipsoidal defect in an elastic body from the results of a single test for uniaxial tension (compression)]. *Izv. RAN. MTT*. 2010, No. 3, pp. 131-142.
5. Alessandrini G., Morassi A., Rosset E. Detecting an inclusion ia an elastic body by boundary measurements. *SIAM J. Math. Anal.* 2002, vol. 33, No. 6, pp. 1247-1268.
6. Morassi A., Rosset E. Detecting rigid inclusions, or cavities, in an elastic body. *J. Elast.* 2003, vol. 73, No. 1-3, pp. 101-126.
7. Vatul'yan A.O., Kashtal'yan D.O. Ob opredelenii lokalizovannoi zony destruktsii v uprugoi balke [On the determination of a localized zone of destruction in an elastic beam]. *Izv. vuzov. Sev.-Kavk. region. Estestv. nauki.* 2015, No. 4, pp. 29-34.
8. Alfutov N.A. *Osnovy rascheta na ustoichivost' uprugikh sistem* [Basics of calculating the stability of elastic systems]. Moscow: Mashinostroenie, 1978, 312 p.
9. Filippov A.P. *Kolebaniya deformiruemykh sistem* [Oscillations of deformable systems]. 2nd ed. Moscow: Mashinostroenie, 1970, 733 p.
10. Amosov A.A., Dubinskii Yu.A., Kopchenova N.V. *Vychislitel'nye metody dlya inzhenerov* [Computational methods for engineers]. Textbook. Moscow: Vyssh. shkola, 1994, 544 p.

Received ***February 15, 2018***

UDC 534.232 DOI 10.23683/0321-3005-2018-2-23-29

ANALISYS OF CONTACT INTERACTION OF PIEZOELASTIC ACTUATOR AND ELASTIC LAYER IN TERMS OF STEADY OSCILLATIONS ON BASIS OF PIN-FORCED METHOD

***A.A. Lyapin*[1]**

[1]*Southern Federal University, Rostov-on-Don, Russia*

Alexander A. Lyapin - Candidate of Physics and Mathema- tics, Senior Researcher, Laboratory of Mechanics of Deformable Bodies and Constructions, Vorovich Institute of Mathematics, Mechanics and Computer Sciences, Southern Federal University, Stachki Ave., 200/1, Rostov-on-Don, 344090, Russia, e-mail: lyapin@sfedu.ru

Application of piezoelastic actuators for generation of acoustic signal while scanning elastic bodies is one of the basic methodic of non-destructive testing. Mathematical modeling for interaction processes of actuator and media allows developing more accurate and modern methods for health monitoring of elastic bodies. In terms of paper presented the calculation of contact interaction of piezoelastic actuator and elastic layer in case of axisymmetric steady-state oscillations is performed. The solution is constructed on the basis of engineering approach, based on description of an actuator as concentrated forces placed along the contact edge and the finite element method for simulation the full problem of an actuator and a media interaction. While performing the finite element simulation the special methodic for dealing with infinitive bodies is used. The correctness for such wave field solutions is checked with known solutions for wave distribution in an elastic layer based on the inverse Hankel integral transformation. Amplitudes for concentrated forces are determined by integration of normal and tangential contact stresses along the contact area. Resulted wave fields are compared for various vibration frequencies.

Keywords: *non-destructive testing, piezoelastic actuator, guided waves, finite element method, pin-force method.*

References

1. Nasedkin A.V., Shikhman V.M., Zakharova S.V. Konechno-elementnyi raschet vysokotemperaturnykh preobrazovatelei akusticheskoi emissii [Finite-element calculation of high-temperature acoustic emission transducers]. *Defektoskopiya*. 2011, No. 7, pp. 49-64.
2. Glushkov E.V., Glushkova N.V., Evdokimov A.A. Distribution of the energy of a piezoelectric actuator between traveling waves excited in an elastic layer. *J. of Applied Mechanics and Technical Physics*. 2015, vol. 56, No. 6, pp. 1007-1014.
3. Glushkov Y.V., Glushkova N.V., Seemann W., Kvasha O.V. The excitation of waves by piezoelectric patch actuators arranged symmetrically on both surfaces of an elastic layer. *J. of Applied Mathematics and Mechanics.* 2011, vol. 75, No. 1, pp. 56-64.
4. Crawley E.F., de Luis J. Use of Piezoelectric Actuators as Elements of Intelligent Structures. *AIAA J.* 1987, vol. 25, No. 10, pp. 1373-1385.
5. Sirohi J., Chopra In. Fundamental Understanding of Piezoelectric Strain Sensors. *J. of Intelligent Material Systems and Structures.* 2000, vol. 11, pp. 246-257.
6. Wood R.J., Steltz E., Fearing R.S. Optimal energy density piezoelectric bending actuators. *Sensors and Actuators A: Physical.* 2005, vol. 119, No. 2, pp. 476-488.
7. Parton V.Z., Kudryavtsev B.A. *Elektromagnitouprugost' p'ezoelektricheskikh i elektroprovodnykh tel* [Electromagnetoelasticity of piezoelectric and electrically conductive bodies]. Moscow: Nauka, 1988, 470 p.
8. Lee C.K., Moon F.C. Laminated piezopolymer plates for torsion and bending sensors and actuators. *J. Acoust. Soc. Am.* 1989, vol. 85, pp. 2432-2439.
9. Tzou H.S., Gader M. Theretical analysis of a multi-layered thin shell coupled with piezoelectric shell actuators for disturbed vibration control. *J. Sound Vibrat.* 1989, vol. 132, pp. 433-450.
10. Crawley E.F., Lazarus K.B. Induced strain actuation of isotropic and anisotropic plates. *AIAA J.* 1991, vol. 29, pp. 944-951.
11. Detwiler D.T., Shen M.-H.H., Venkayya V.B. Finite element analysis of laminated composite structures containing distributed piezoelectric actuators and sensors. *Finite Elements in Analysis and Design.* 1995, vol. 20, No. 2, pp. 87-100.
12. Sung Yi, Shih Fu Ling, Ming Ying. Large deformation finite element analyses of composite structures integrated with piezoelectric sensors and actuators. *Finite Elements in Analysis and Design.* 2000, vol. 35, No. 1, pp. 1-15.
13. Giurgiutiu V. *Structural Health Monitoring with Piezoelectric Wafer Active Sensors*. Academic Press: Burlington, MA, USA, 2007, 1024 p.

Received ***February 15, 2018***

UDC 513.98 DOI 10.23683/0321-3005-2018-2-30-34

ON THE CONVERGENCE OF THE DIFFERENCE METHOD FOR SOLVING THE CAUCHY PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION WITH THE RIEMANN-LIOUVILLE FRACTIONAL DIFFERENTIATION OPERATOR

F.F. Malieva[1], V.D. Beybalaev[1,2]
[1]Dagestan State University, Makhachkala, Russia,
[2]Dagestan State University of National Economy, Makhachkala, Russia

Farida F. Malieva - Postgraduate, Department of Applied Mathematics, Dagestan State University, Gadzhieva St., 43a, Makhachkala, Republic Dagestan, 367000, Russia, e-mail: faridadavudova@mail.ru

Vetlugin D. Beybalaev - Candidate of Physics and Mathematics, Associate Professor, Department of Applied Mathematics, Dagestan State University, Gadzhieva St., 43a, Makhachkala, Republic Dagestan, 367000, Russia; Associate Professor, Department of Mathematics, Dagestan State University of National Economy, Ataeva St., 5, Makhachkala, Republic Dagestan, 3670008, Russia, e-mail: kaspij_03@mail.ru

In this paper, we study the difference method of order (2-α) *of the accuracy of the solution of the Cauchy problem for a differential equation with the Riemann-Liouville fractional differentiation operator. A difference approximation is obtained for the fractional Riemann-Liouville derivative of order* (2-α) *of accuracy. It is shown that the difference scheme approximates the differential equation* (2-α) *by the order of accuracy. It is shown that the difference scheme approximates the differential equation* (2-α) *by the order of accuracy, and the sum of the coefficients is less than or equal to one. An error estimate is obtained for the proposed numerical method for solving the Cauchy problem. For the error of the method under the conditions that if the right-hand side of the equation satisfies the Lipschitz condition with constant* A *in the second argument and* $q = A\tau^{\alpha}\Gamma(1-\alpha) < 1$, *an estimate is obtained from which the convergence of the proposed numerical method for solving the Cauchy problem for the ordinary differential equation with the Riemann-Liouville fractional differentiation operator follows. The proposed numerical method for solving the Cauchy problem has a higher accuracy order than the numerical methods proposed earlier.*

Keywords: *fractional derivative, approximation, difference scheme, stability, convergence, differential equations.*

References

1. Meerschaert M.M., Tadjeran C. Finite difference approximations for two-sides space-fractional partial differential equations. Applied Numerical Mathematics. 2006, vol. 56, pp. 80-90.
2. Abduragimov E.I., Omarova R.A. Polozhitel'noe reshenie granichnoi zadachi dlya odnogo nelineinogo differentsial'nogo uravneniya s drobnymi proizvodnymi [Positive solution of the boundary value problem for one nonlinear differential equation with fractional derivatives]. *Vestn. DGU.* 2015, iss. 6, pp. 99-104.
3. Beibalaev V.D. Matematicheskaya model' perenosa v sredakh s fraktal'noi strukturoi [Mathematical model of transfer in mediums with fractal structure]. *Mat. modelirovanie.* 2009, vol. 21, No. 5, pp. 55-62.
4. Nagorov A.L. [Numerical methods for fractional operators in image processing systems]. *Fundamental'nye i prikladnye problemy matematiki i informatiki* [Fundamental and applied problems of mathematics and Informatics]. Proceedings of the Conference. Makhachkala, 2017, pp. 138-140.
5. Nazaraliev M.A., Beibalaev V.D. Chislennye metody resheniya kraevoi zadachi dlya uravneniya teploperenosa s proizvodnoi drobnogo poryadka [Numerical methods for solving the boundary value problem for heat transfer equation with derivative of fractional order]. *Vestn. DGU.* 2008, iss. 6, pp. 46-53.
6. Beibalaev V.D. Raznostnye metody resheniya zadachi Koshi dlya obyknovennogo differentsial'nogo uravneniya s operatoram drobnogo differentsirovaniya [Difference methods for solving the Cauchy problem for an ordinary differential equation with fractional differentiation operators]. *Vestn. DGU.* 2014, iss. 6, pp. 53-61.
7. Beibalaev V.D. Chislennyi metod resheniya zadachi perenosa s dvustoronnei proizvodnoi drobnogo poryadka [Numerical method of solving the problem of transferring from two-sided derivative of fractional order]. *Vestn. SamGTU. Fiz.-mat. nauki*. 2009, No. 1 (18), pp. 267-270.
8. Beibalaev V.D. Chislennyi metod resheniya matematicheskoi modeli teploperenosa v sredakh s fraktal'noi strukturoi [Numerical solution method the mathematical model of heat transfer in media with fractal structure]. *Fundamental'noe issledovanie*. 2007, No. 12, pp. 67-71.

9. Samko S.G., Kilbas A.A., Marichev O.I. *Integral'nye i proizvodnye drobnogo poryadka i nekotorye ikh prilozheniya* [Integral and fractional order derivatives and some of their applications]. Minsk: Nauka i tekhnika, 1987, 498 p.

10. Nakhushev A.M. *Elementy drobnogo ischisleniya i ikh primenenie* [Elements of fractional calculus and their application]. Nalchik, 2003, 299 p.

11. Samarskii A.A., Gulin A.V. *Chislennye metody* [Numerical methods]. Moscow: Nauka, 1989, 430 p.

Received ***December 18, 2017***

UDC 534.1 DOI 10.23683/0321-3005-2018-2-35-44

DYNAMICS AND STABILITY OF ELECTROSTATIC TRANSDUCER UNDER THE INFLUENCE OF HEAT IMPULSE

***N.F. Morozov*[1], *D.A. Indeitsev*[1,2,3], *A.V. Lukin*[2], *I.A. Popov*[2], *O.V. Privalova*[2], *L.V. Shtukin*[2,3]**

[1]*St. Petersburg State University, St. Petersburg, Russia,*
[2]*Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia,*
[3]*Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg, Russia*

Nikita F. Morozov - Doctor of Physics and Mathematics, Professor, Academician, Russian Academy of Sciences, Head of the Department of Elasticity Theory, Faculty of Mathematics and Mechanics, St. Petersburg State University, Universitetskii Pr., 28, Peterhof, St. Petersburg, 198504, Russia, e-mail: n.morozov@spbu.ru

Dmitry A. Indeitsev - Doctor of Physics and Mathematics, Professor, Corresponding Member, Russian Academy of Sciences, Department of Elasticity Theory, Faculty of Mathematics and Mechanics, St. Petersburg State University, Universitetskii Pr., 28, Peterhof, St. Petersburg, 198504, Russia; Scientific Advisor, Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, Bolshoi Ave., 61, St. Petersburg, 199178, Russia; Head of the Department of Mechanics and Control Processes, Institute of Mathematics and Mechanics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia, e-mail: dmitry.indeitsev@gmail.com

Aleksey V. Lukin - Postgraduate, Assistant, Department of Mechanics and Control Processes, Institute of Mathematics and Mechanics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia, e-mail: lukin_av@spbstu.ru

Ivan A. Popov - Postgraduate, Assistant, Department of Mechanics and Control Processes, Institute of Mathematics and Mechanics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia, e-mail: popov_ia@spbstu.ru

Olga V. Privalova - Candidate of Technical Sciences, Associate Professor, Department of Mechanics and Control Processes, Institute of Mathematics and Mechanics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia, e-mail: o.privalova@mail.ru

Lev V. Shtukin - Candidate of Physics and Mathematics, Associate Professor, Department of Mechanics and Control Processes, Institute of Mathematics and Mechanics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia; Senior Researcher, Laboratory for Mathematical Modelling of Wave Phenomena, Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, Bolshoi Ave., 61, St. Petersburg, 199178, Russia, e-mail: Lvtvsh4749@gmail.com

Laser-induced vibrations and elastic stability of a clamped-clamped beam electrostatic transducer are considered under ultrafast laser pulse. It is assumed that laser pulse acts as volume heat generation with Gaussian time-profile localized in near-surface layer of the beam. Temperature load non-stationarity and non-homogeneity through length and thickness lead to appearance of thermal-induced mechanical moment and axial forced acting on the beam, which can result in buckling phenomena. Semi-analytical methods for solution of nonlinear boundary-value problems are used for static equilibrium determination of the beam in the electric field of one stationary electrode. Analytical solution of non-stationary temperature problem in the beam volume is obtained. Finally, areas in parameter space of system geometrical and mechanical properties along with laser pulse characteristics are determined which correspond to elastic stability of initial equilibrium form of the beam subjected to laser pulse.

Keywords: *elastic stability, laser-induced vibrations, microelectromechanical systems, coupled-field problems, dynamics.*

References

1. Lavrent'ev M.A., Ishlinskii A.Yu. Dinamicheskie formy poteri ustoichivosti uprugikh sistem [Dynamic forms of loss of stability of elastic systems]. *Dokl. AN SSSR.* 1949, vol. 64, No. 6, pp. 776-782.
2. Vol'mir A.S. *Nelineinaya dinamika plastinok i obolochek* [Nonlinear dynamics of plates and shells]. Moscow: Nauka, 1972, 432 p.
3. Morozov N.F., Tovstik P.E. O dinamicheskoi potere ustoichivosti sterzhnya pri prodol'noi nagruzke, men'shei eilerovoi [On the dynamic loss of stability of the rod at a longitudinal load less than the Eulerian]. *Dokl. RAN.* 2013, vol. 453, No. 3, pp. 282-285.
4. Belyaev A.K., Morozov N.F., Tovstik P.E., Tovstik T.P. Parametricheskie rezonansy v zadache o prodol'nom udare po tonkomu sterzhnyu [Parametric resonance in the problem of longitudinal impact on a thin rod]. *Vestn. SPbGU. Matematika. Mekhanika. Astronomiya.* 2016, vol. 3 (61), iss. 1, pp. 77-94.
5. Poletkin K., Kulish V. [et al.] Thermal properties of thin films studied by ultrafast laser spectroscopy: Theory and experiment. *International J. of Heat and Mass Transfer*. 2014, April, pp. 1-20.
6. Jx Ma [et al.] Analytical solution of transient heat conduction in bi-layered circular plate irradiated by laser pulse. *Canadian J. of Physics.* 2016, vol. 95 (4), pp. 322-330.
7. Xinwei Wang, Xiafan Xu. Thermoelastic waves in metal induced by ultrafast laser pulses. *J. of Thermal Stresses.* 2002, vol. 25, pp. 457-473.
8. Younnis M.I. *MEMS Linear and Nonlinear Statics and Dynamics*. Springer Science, 2011, 453 p.
9. Evstifeev M.I., Chelpanov I.B. Voprosy obespecheniya stoikosti mikromekhanicheskikh giroskopov pri mekhanicheskikh vozdeistviyakh [The problems of providing of stability of MEMS gyroscopes the mechanical influences]. *Giroskopiya i navigatsiya*. 2013, No. 1 (80), pp. 119-133.
10. Nekrasov Ya.A., Moiseev N.V., Belyaev Ya.V., Pavlova S.V., Lokshonkov R.G. Vliyanie postupatel'nykh vibratsii, udarov i akusticheskikh pomekh na kharakteristiki mikromekhanicheskogo giroskopa [The influence of translational vibration, shock and acoustic noise on the characteristics of micromechanical gyroscope]. *Giroskopiya i navigatsiya*. 2016, No. 2 (93), pp. 56-67.
11. Jx Ma [et al.] Thermoelastic response of a simply supported beam irradiated by a movable laser pulse. *Canadian J. of Physics*. 2017, vol. 95 (10), pp. 1-11.
12. Nayfeh A.H., Pai P.F. *Linear and Nonlinear Structural Mechanics*. New York: Wiley, 2004, 746 p.
13. Tang D.W., Araki N. Wavy, wavelike, diffusive thermal responses of finite rigid slabs to high-speed heating of laser pulses. *International J. of Heat and Mass Transfer*. 1999, vol. 42, pp. 855-860.
14. Daining Fang, Soh A.K. Laser-induced vibrations of microbeams under different boundary conditions. *International J. of Solids and Structures.* 2008, vol. 45, pp. 1993-2013.
15. Novatskii V. *Dinamicheskie zadachi termouprugosti* [Dynamic problems of thermoelasticity]. Moscow: Mir, 1970, 256 p.
16. Wen-Ming Zhang [et al.]. Electrostatic pull-in instability in MEMS/NEMS: A review. *Sensors and Actuators A.* 2014, vol. 214, pp. 187-218.
17. Lukin A.V., Popov I.A., Skubov D.Yu. [Investigation of stability and branching of equilibrium forms of elastic elements of microsystems technology]. *Nelineinaya dinamika mashin – SCHOOL-NDM 2017* [Nonlinear dynamics of machinery – SCHOOL-NDM 2017]. Moscow, 2017, pp. 313-322.
18. Lukin A.V., Popov I.A., Skubov D.Yu. Nelineinaya dinamika i ustoichivost' elementov mikrosistemnoi tekhniki [Nonlinear dynamics and stability of elements of microsystem technique]. *Nauch.-tekhn. vestn. informatsionnykh tekhnologii, mekhaniki i optiki*. 2017, vol. 17, No. 6, pp. 1107-1115.
19. Vladimirov V.S. *Sbornik zadach po uravneniyam matematicheskoi fiziki* [Collection of problems on equations of mathematical physics]. 5th ed. Moscow: Fizmatlit, 2016, 520 p.
20. Nayfeh A.H., Balachandran B. *Applied Nonlinear Dynamics: Analytical, Computational and Experimental Methods.* Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA, 2004, 685 p.
21. Kuznetsov Yu.A. *Elements of Applied Bifurcation Theory*. New York: Springer Verlag, 2004, 631 p.
22. Govaerts W., Kuznetsov Yu.A. [et al.]. *MATCONT and CL_MATCONT: Continuation toolboxes in MATLAB.* Utrecht University, 2018, 128 p.
23. Demidovich B.P. *Lektsii po teorii ustoichivosti dvizheniya* [Lectures on the theory of stability of motion]. Moscow: Nauka, 1967, 472 p.
24. Verhulst F. *Nonlinear Differential Equations and Dynamical Systems*. Springer Verlag, 1990, 277 p.

Received ***April 12, 2018***